Nuclear Science
2020; 5(1): 1-7
http://www.sciencepublishinggroup.com/j/ns
doi: 10.11648/j.ns.20200501.11
ISSN: 2640-4338 (Print); ISSN: 2640-4346 (Online)

A Dispersive Optical Model Analysis of the Neutrons Scattering by Titanium Element Nucleus and Its Natural Isotopes

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To cite this article:
Haiddar Al-Mustafa, Anees Belal. A Dispersive Optical Model Analysis of the Neutrons Scattering by Titanium Element Nucleus and Its Natural Isotopes. *Nuclear Science*. Vol. 5, No. 1, 2020, pp. 1-7. doi: 10.11648/j.ns.20200501.11

Received: December 26, 2019; **Accepted**: March 6, 2020; **Published**: April 30, 2020

Abstract: In this paper, a dispersive optical model analysis of the neutrons scattering by titanium element nucleus and its natural isotopes is applied to the construction of the complex single-particle mean field starting from Fermi energy value to the energy value 100 MeV and for constant input values of the parameters of this mean field and the varied input values of Hatree-Fock approximation parameters of the nonlocal potential. The results according to DOMACNIP program that has been designed for that purpose would contain: continuous energy variation of the depths of the real and imaginary parts of the mean field, which are connected by dispersion relations were compared with these resulting from global parameterization of the optical model potential. In addition to continuous energy variation of the real radius parameter of the Wood-Saxon approximation to the mean field potential with its Hatree-Fock approximation of the nonlocal potential. Consequently, our results for the continuous energy variations of the predicted (total, total reaction, elastic) cross sections within the energy range (1-100) MeV, and with calculation step of the pervious range whose magnitude (1 MeV), elastic differential cross section and polarization for selected energy and for selected center-of-mass scattering angle within the energy range (1-100) MeV showed the excellent agreement with available experimental data and better than these resulted from global parameterization of the optical model potential, and thus more reliable for calculation the cross sections of unknown interactions of elements nuclei and their isotopes such neutrons scattering by titanium element nucleus and its natural isotopes.

Keywords: Dispersive Optical Model Analysis (DOMA), Neutrons Scattering, Hatree-fock Potential, Dispersion Relations (DR), Mean Field, Fermi Energy, Cross Section, Polarization

1. Introduction

The nuclear optical model potential is of the fundamental importance concepts in the nuclear physics. It describes the motion of one nucleon, bound or unbound, in the mean field of all the other nucleons comprising the nucleus. The field due to the sum of all the individual nucleon-nucleon interactions is thus represented by a simple one-body potential. This approximation greatly simplifies the calculation of a wide range of nuclear structure and nuclear reaction phenomena, in addition to the excellent agreement with experimental data [1]. The application of the concept of the nuclear mean field is for understanding the properties of bound single-particle states and for elastic scattering of unbound nucleons [1-3].

The phenomenological optical model potential for nucleon-nucleus scattering, U, is defined as [2-6]:

$$U(r,E) = -V_V(r,E) - V_{SO}(r,E).\vec{\sigma}.\vec{l} + V_S(r,E) + V_C(r) + i\left(-W_V(r,E) - W_S(r,E) + W_{SO}(r,E).\vec{\sigma}.\vec{l}\right) \quad (1)$$

Where $V_{V,S}$ and $W_{V,S,SO}$ are the real and imaginary components of the volume-central (V), surface-central (S) and spin-orbit (SO) potentials, respectively. E is the LAB energy of the incident particle in MeV. All components are separated in energy-dependent well depths, V_V, V_S, W_V, W_S

and W_{SO}, and energy-independent radial parts f, namely

$$\begin{aligned} V_V(r,E) &= V_V(E) f(r,R_V,a_V) \\ W_V(r,E) &= W_V(E) f(r,R_V,a_V) \\ W_S(r,E) &= -4a_S W_S(E) \frac{d}{dr} f(r,R_S,a_S) \\ V_S(r,E) &= -4a_S V_S(E) \frac{d}{dr} f(r,R_S,a_S) \\ V_{SO}(r,E) &= V_{SO}(E) (\frac{\hbar}{m_\pi c})^2 \frac{1}{r}\frac{d}{dr} f(r,R_{SO},a_{SO}) \\ W_{SO}(r,E) &= W_{SO}(E) (\frac{\hbar}{m_\pi c})^2 \frac{1}{r}\frac{d}{dr} f(r,R_{SO},a_{SO}) \end{aligned} \quad (2)$$

The form factor $f(r, R_i, a_i)$ is a Wood-Saxon shape

$$f(r, R_i, a_i) = \frac{1}{[1+e^{(\frac{r-R_i}{a_i})}]} \quad (3)$$

Where the geometry parameters are the radius $R_i = r_i A^{\frac{1}{3}}$,, with A the atomic mass number, and the diffuseness parameters $a_i, i = V, SO, S$. For neutrons scattering, the value of the coulomb term V_C, is zero.

By solving the Schrödinger equation numerically with this complex potential yields a wealth of valuable information; it returns a prediction for the basic observables, namely the elastic angular distribution and polarization, the reaction and total cross section, and the detailed information of the calculation methodology that is showed in the reference [6].

The essential value of a good optical model is that it can reliably predict these quantities for energies and nuclides for which no measurements exist as is the state of the neutrons scattering by titanium element nucleus and its natural isotopes. Also, the quality of the not directly observable quantities that are provided by the optical model has an equally important impact on the evaluation of the various reaction channels.

The dispersive optical model analysis describes the continuous energy variation of the nuclear mean field potential components depths and connection between the real parts and imaginary parts of the mean field by a dispersion relation, and so the reliable determination of the mean field is perfect by comparing a prediction of the cross sections with these are measured experimentally.

There are many published studies for detailed analyses of data for the neutron scattering state, some of these studies depended on the single fits of the experimental data and others depended on dispersion relations. In both states our dependence is on global parametrization of the optical model potential which agree with the energy and atomic mass ranges of the titanium element nucleus and its natural isotopes.

The present paper aims at presenting the dispersive optical model analysis (DOMA) of the neutrons scattering by titanium element nucleus and its natural isotopes and comparing the results with these resulted from global parametrization of the optical model potential and averrable experimental data within energy range (1-100) MeV and with calculation step of the previous range whose magnitude 1 MeV, according to evaluated fitting methodology.

2. Methodology

The methodology of a dispersive optical model analysis is similar to the proton scattering that is showed in references [7, 8], but because of; unavailable experimental data of the cross sections of the neutrons scattering by titanium element nucleus and its natural isotopes except only one value of the total cross section for the energy value (14.2 MeV), in addition to the variations of the total cross section value at this value of the energy for each isotope from the titanium isotopes, we have depended on the follow fitting methodology:

i. The values of the input parameters for Brown-Rho and mean field parameters have been constant for the titanium element nucleus and its natural isotopes.
ii. The values of the input parameters for Hatree-Fock term have been varied for each isotope from the titanium isotopes, whereas for titanium element nucleus have been got as sum of Hatree-Fock parameter values of each isotope multiplied with the relative abundance of this isotope as they are showed in the table 1.
iii. After calculating the depths, the geometrical parameters and the volume integral per nucleon of the mean field components, we have compared them with global parameterizations of the optical potential whose calculations have been performed in the DOMACNIP program:

1. Becchetti and Greenlees [9], its coding in the program BG, for

$$E \leq 50 \text{ Mev}, Z_t = (20-82), A_t = (40-238)$$

2. Varner et al [10], its coding in the program CH, for

$$10 \leq E \leq 26 \text{ Mev}, Z_t = (20-83), A_t = (40-209)$$

3. Koning and Delaroche [2], its coding in the program KD, for

$$0.001 \leq E \leq 200 \text{ Mev}, Z_t = (12-83), A_t = (24-209)$$

While the values of a spin-orbit coupling term of the mean field in our calculations are:

$$V_{SO} = V_{SO_{KD}}, W_{SO} = 0, r_{SO} = r_{SO_{KD}}, a_{SO} = a_{SO_{KD}}$$

Also, we have compared our results with these global parameterizations for the continuous energy variations of the predicted (total, total reaction, elastic) cross sections, elastic differential cross section and polarization for selected center-of-mass scattering angle within the energy range (1-100) MeV and with calculation step of the pervious range whose magnitude (1 MeV), in addition to elastic differential cross section and polarization for the energy value (14.2 MeV) and within the angular range of the center-of-mass scattering angle $(5° - 175°)$ and with calculation step of the pervious range whose magnitude (5°).

3. Results and Discussion

The results According to DOMACNIP program are summarized as follows:

3.1. Input Parameters

The values of the constant and changed input parameters for the neutrons scattering by titanium element nucleus and its natural isotopes are showed in the (Table 1).

3.2. Depths of the Mean Field

The depths of the mean field are compared with these resulted from global parameterizations of the optical potential within the energy range $(1-100)$MeV and with calculation step of the previous range whose magnitude 1 MeV, as they are showed in the Figures (1-3).

3.3. The Real Radius Parameter of the Mean Field

The real radius parameter of the Wood-Saxon approximation to the mean field potential with its HF approximation, within the energy range $(E_f - 100)$ MeV and with calculation step of the previous range whose magnitude 1 MeV, as it is showed in the Figure 4.

Table 1. *The input values of the constant and changed parameter.*

Constant Input Parameters						
Brown-Rho Parameters			**Geometrical Parameters Volume and Surface Absorption**			
ρ_w, **MeV**	ρ_{W_V}, **MeV**	β, **MeV.fm**^{3}	r_{wv}, **fm**	r_{ws}, **fm**	a_{wv}, **fm**	a_{ws}, **fm**
2.5	24.2	-87.4	1.28	1.28	0.69	0.62

Diffuseness Parameter of the Hartree-Fock Field	**Diffuseness Parameter of the real part from the mean field**
a_{HF}, **fm**	a_V, **fm**
0.7	0.7

(Projectile-Target) Parameters			**Maximum Energy**
Z_P	A_P **(amu)**	Z_t	E_{Lab}**(Max) MeV**
0.0	1.0086	22	100

Changed Input Parameters						
Titanium nucleus Ti (Na)		**Natural Titanium Isotopes**				
		Ti − 46	**Ti − 47**	**Ti − 48**	**Ti − 49**	**Ti − 50**
A_t (amu) [11-14]	47.867	45.9526	46.9518	47.948	48.9479	49.9448
E_f (MeV)	-9.8642	-10.9744	-10.1930	-9.8241	-9.4803	-8.5954
Hartree-Fock Parameters						
r_{HF}, fm	1.212	1.225	1.2	1.22	1.19	1.18
α_{HF}	0.386	0.37	0.38	0.39	0.37	0.38
$\mathcal{V}_{HF}(E_F)$, MeV	-51.9710	-52.1	-52.4	-52.1	-52.2	-51.8
Relative Abundance [13-14]		8.25%	7.44%	73.72%	5.41%	5.18%
Experimental value at (E_{Lab} = 14.2 MeV), mb [15]		2454 ± 35	2319 ± 48	2492 ± 25	2417 ± 41	2393 ± 63

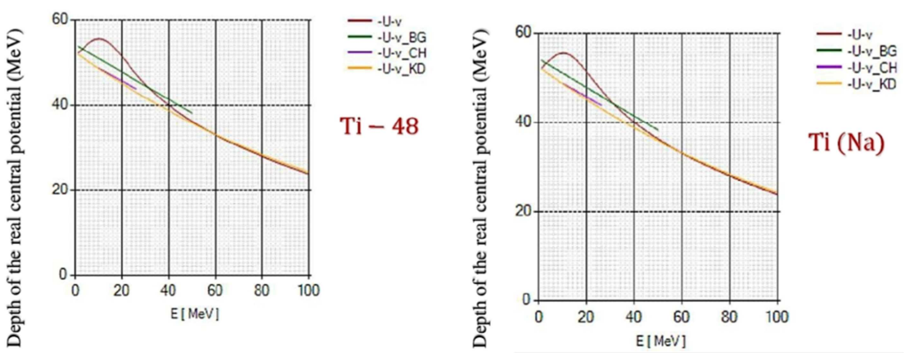

Figure 1. *Depth of the real part of the mean field potential as a function of neutron energy (the red line) compared with these resulted from global parametrization of the optical model potential.*

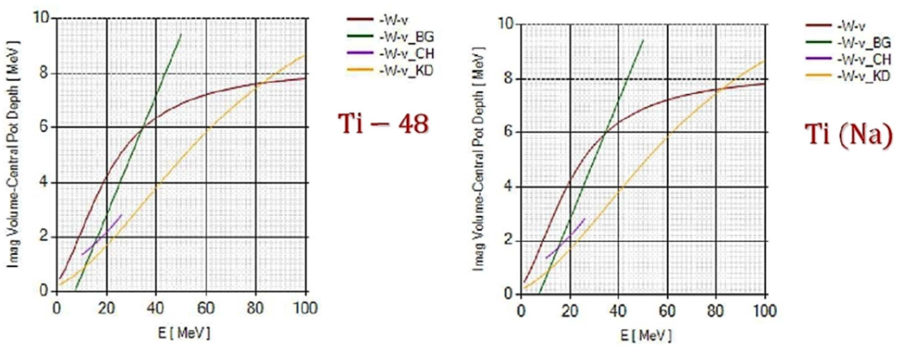

Figure 2. *Depth of the volume component of the imaginary part of the mean field mean field potential as a function of neutron energy (the red line) compared with these resulted from global parametrization of the optical model potential.*

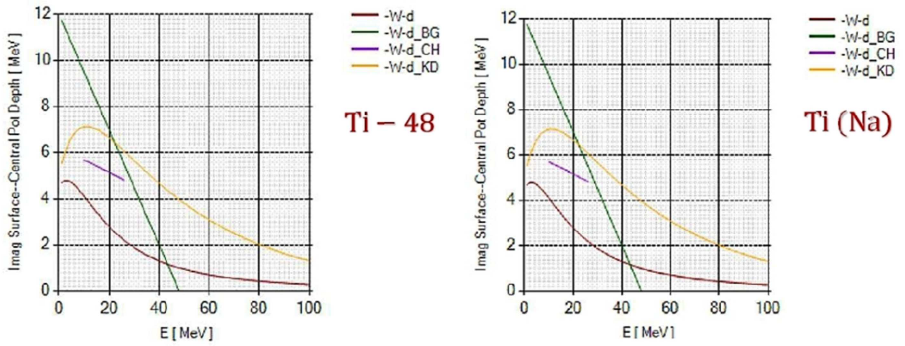

Figure 3. *Depth of the surface-peaked component of the imaginary part of the mean field mean field potential as a function of neutron energy (the red line) compared with these resulted from global parametrization of the optical model potential.*

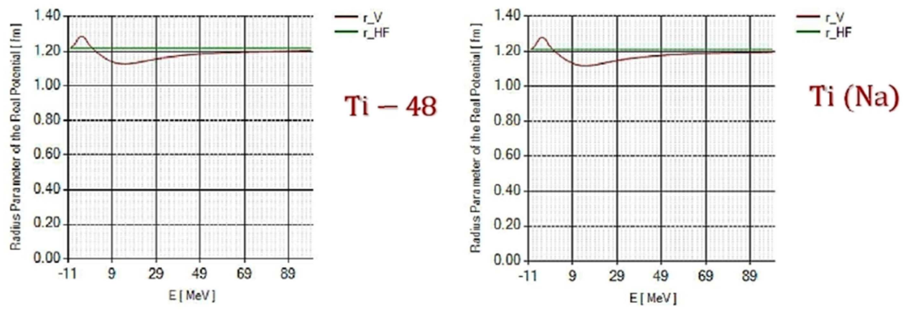

Figure 4. *The energy dependence of the radius parameter of the Wood-Saxon approximation to the mean field potential with its HF approximation.*

3.4. Cross Sections

The (total, total reaction, elastic) cross sections within the energy range $(1-100)$ MeV and with calculation step of the pervious range whose magnitude 1 MeV are compared with these resulted from global parameterizations of the optical potential and with available experimental data, and are in ($\mathrm{fm}^2 = 10$ mb), as it is showed in the Figures 5-7. There is excellent agreement with the experimental data and the global parametrization of the optical potential according to our calculations.

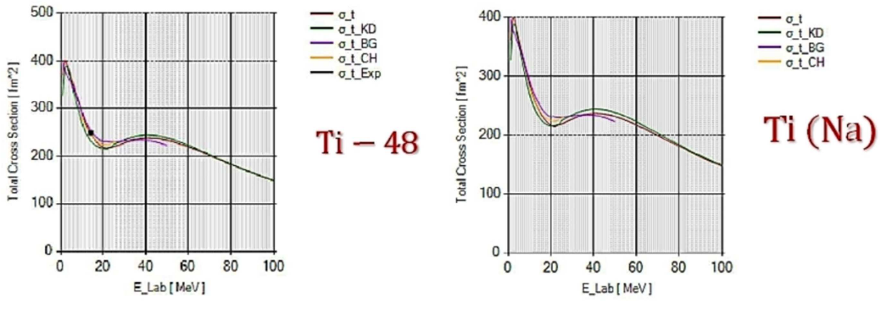

Figure 5. *The energy dependence of the $(n + {}^{(48,natural)}Ti)$ total cross section (the red line) compared with experimental value and with these resulted from global parametrization of the optical model potential.*

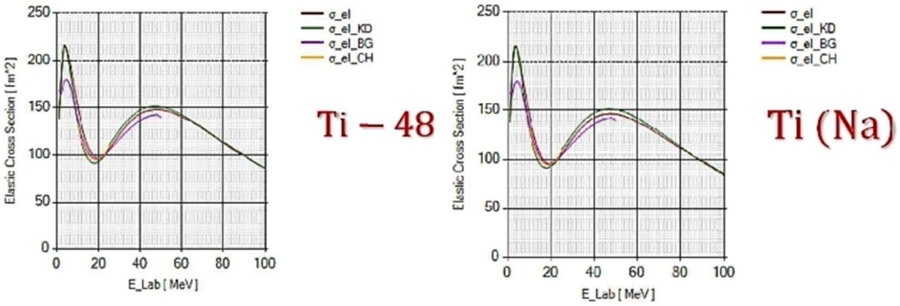

Figure 6. The energy dependence of the $(n + {}^{(48,natural)}Ti)$ elastic cross section (the red line) compared with experimental value and with these resulted from global parametrization of the optical model potential.

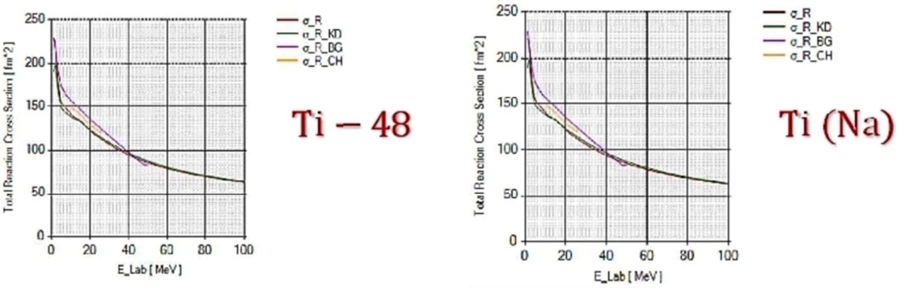

Figure 7. The energy dependence of the $(n + {}^{(48,natural)}Ti)$ total reaction cross section (the red line) compared with experimental value and with these resulted from global parametrization of the optical model potential.

3.5. Elastic Differential Cross Sections and Polarization for Selected Energy

The elastic differential cross sections and polarization for selected energy whose magnitude (14.2)MeV compared with these resulted from global parameterizations of the optical potential, as which are showed in the Figures 8-9. There is an excellent agreement with the global parametrization of the optical model potential according to our calculations.

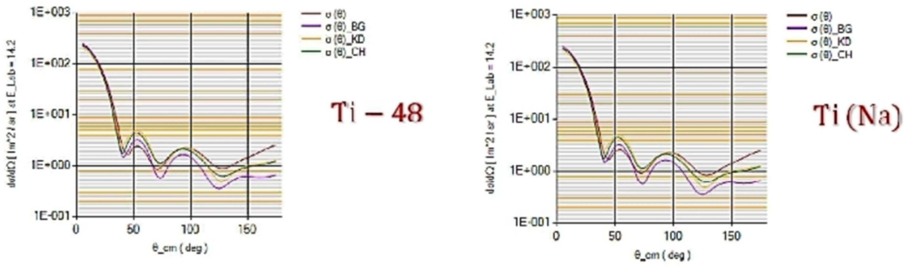

Figure 8. Dependence of the $(n + {}^{(48,natural)}Ti)$ elastic differential cross section upon the center-of-mass scattering angle (the red line) compared with these resulted from global parametrization of the optical model potential, for $E_{Lab} = 14.2\ MeV$.

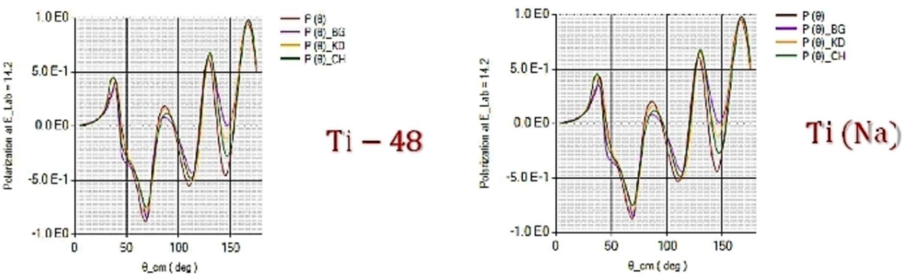

Figure 9. Dependence of the $(n + {}^{(48,natural)}Ti)$ polarization upon the center-of-mass scattering angle (the red line) compared with these resulted from global parametrization of the optical model potential, for $E_{Lab} = 14.2\ MeV$.

3.6. Elastic Differential Cross Sections and Polarization for Selected Angle

The elastic differential cross sections and polarization for selected angle whose magnitude $\theta_{cm} = 30°$ compared with these resulted from global parameterizations of the optical potential, as which are showed in the Figures 10-11. There is an excellent agreement with the global parametrization of the optical model potential according to our calculations.

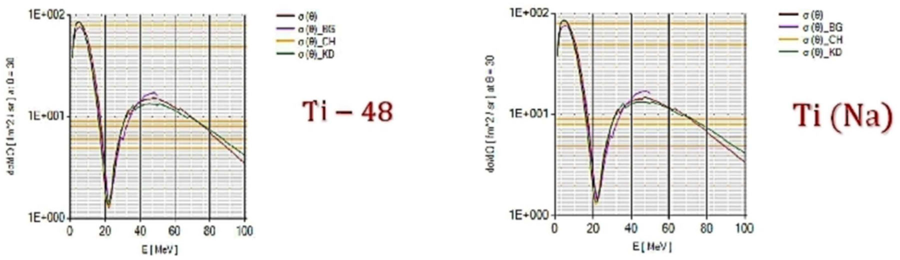

Figure 10. *The energy dependence of the* $(n + {}^{(48,natural)}Ti)$ *elastic differential cross section (the red line) compared with these resulted from global parametrization of the optical model potential, for* $\theta_{cm} = 30°$.

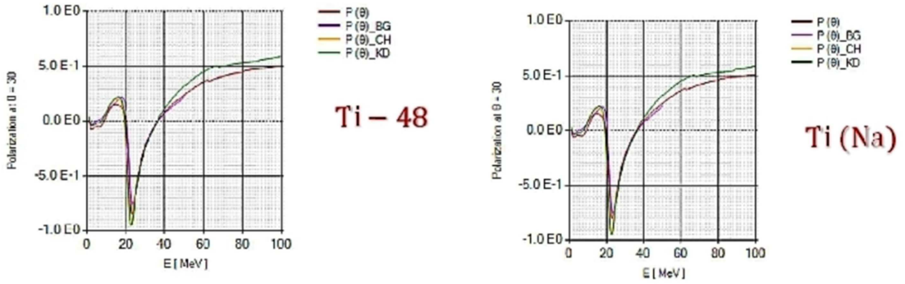

Figure 11. *The energy dependence of the* $(n + {}^{(48,natural)}Ti)$ *polarization (the red line) compared with these resulted from global parametrization of the optical model potential, for* $\theta_{cm} = 30°$.

4. Conclusion

The important conclusions can be shown as follows:

i. Our result according to the dispersive optical model analysis of the neutron scattering by titanium element nucleus and its natural isotopes have been drawn for constant input value of the mean field parameters and changed input value of the Hatree-Fock parameters.

ii. Our calculation within the energy range $(1 - 100)$MeV and with calculation step of the pervious range whose magnitude 1 MeV of the continuous energy variation of the depths of the real and imaginary parts of the mean field were compared with these resulting from global parameterization of the optical model potential. In addition to continuous energy variation of the real radius parameter of the Wood-Saxon approximation to the mean field potential with its Hatree-Fock approximation of the nonlocal potential within the energy range $\left(E_f - 100\right)$MeV.

iii. Our prediction of the (total, total reaction, elastic) cross section data within the energy range $(1 - 100)$ MeV showed excellent agreement with available experimental data and the better than these resulted from global parameterization of the optical model potential.

iv. Our prediction of the elastic differential cross section and polarization data for selected energy (14.2 MeV) within the angular range $\theta_{cm} = (5° - 175°)$, and also for selected angle $\theta_{cm} = 30°$ within the energy range $(1 - 100)$ MeV showed excellent agreement with these resulted from global parameterization of the optical model potential.

Depending upon these conclusions, we suggest to extend the work on this methodology to unify of the optical model potential for bound and scattering states of the nucleons (neutron and proton) by scattering by titanium element nucleus and its natural isotopes, whose experimental data are poor or unknown. Therefore, more reliable in analysis and the prediction for experimental data.

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